arXiv:1104.1465v2 [hep-ph] 5 Aug 2011

OU-HET-702

Gauge-independence of gluon spin in the nucleon and its evolution

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In recent papers, we have established the existence of gauge-invariant decomposition of nucleon spin, each term of which can be related to known high-energy deep-inelastic-scattering observables. A subtlety remains, however, for the intrinsic spin part of gluons at the quantum level. In fact, it was sometimes claimed that the evolution of gluon spin depends on the gauge choice and its physical interpretation makes sense only in the light-cone gauge. In the present paper, we will demonstrate explicitly that the gluon spin operator appearing in our decomposition evolves gauge-independently and that it properly reproduces the familiar evolution equation for the 1st moments of polarized quark and gluon distributions obtained with the Altarelli-Parisi method, which cannot directly be checked by the standard operator expansion method.

In analyzing the decomposition of the nucleon spin, color gauge-invariance is still the cause of intense debate [1] -[16]. Although it has long been believed that the total angular momentum of the gluon cannot be gauge-invariantly decomposed into its intrinsic spin and orbital parts [1],[4], this mistaken belief has recently been questioned [9]-[12]. In fact, we now have two different gauge-invariant decompositions of nucleon spin into the spin and orbital angular momenta of quarks and gluons. The one is the decomposition proposed by Chen et al. [9],[10] and the other is the one advocated by the present author [11],[12]. A crucial advantage of the latter decomposition is observability of each term of decomposition by means of known high-energy deep-inelastic-scattering (DIS) measurements [12].

The circumstances for the quark part should already be clear from the pioneering works by Ji [4],[17]. First, the decomposition of the nucleon spin into the quark and gluon total angular momentum, J_q and J_g, is manifestly gauge-invariant, and each term can in principle be extracted from generalized-parton-distribution (GPD) analyses. Since the intrinsic quark spin (or the longitudinal quark polarization) $\Delta\Sigma$ is clearly gauge-invariant and measurable, the quark orbital angular momentum (OAM) defined as $L_q \equiv J_q - \frac{1}{2}\,\Delta\Sigma$ should also be gauge-invariant. An important fact here is that the quark OAM obtained in that way corresponds to a nucleon matrix element of *dynamical* quark OAM operator containing the full gauge-covariant derivative [4],[17], not the *canonical* OAM or its nontrivial gauge-invariant extension as advocated by Chen et al. [9],[10].

Although slightly more delicate, the situation for the gluon part appears quite analogous. It is widely believed that the gluon spin (or the longitudinal gluon polarization) Δg is a measurable quantity from the polarized DIS experiments. Then, if one defines the gluon OAM by $L_g \equiv J_g - \Delta g$, L_g is clearly a measurable quantity. A key question here is whether Δg is really a gauge-invariant quantity or not. If it is indeed the case (as is vaguely anticipated), it is a logical conclusion that L_g is also gauge-invariant. In this sense, a convincing check of the gauge-invariance of gluon polarization in the nucleon is a fundamentally important homework left in the nucleon spin physics. The gauge-invariant gluon spin operator was first proposed by Chen et al. [9],[10], and it was later confirmed by us [11],[12]. Our investigation goes further. We could show in [12] that the gluon OAM L_g defined by $L_g \equiv J_g - \Delta g$, or more precisely the difference between the 2nd moment of GPD $H_g(x,\xi,t) + E_g(x,\xi,t)$ and the 1st moment of the polarized gluon distribution $\Delta g(x)$, coincides with the nucleon matrix element of the *dynamical* gluon OAM containing the *potential angular momentum* term explained in [11], not the canonical OAM [15] or its nontrivial gauge-invariant extension advocated by Chen et al. [9],[10]

In spite of the above-mentioned nice correspondence between the quark and gluon sectors, there still remains a subtlety in the gluon part. It is no wonder that the gluon spin operator given in [12] is gauge-invariant at least formally, or at the classical level. It was also verified in [12] that its nucleon matrix element reduces to the 1st moment of $\Delta g(x)$ in the LC gauge. If this is the case, a general thinking based on the gauge pinciple dictates the following. Since our gluon spin operator (although not necessarily local) is gauge-invariant, its numerical value should be the same also in any other gauges than the LC gauge. Furthermore, its Q^2-evolution should also be gauge-independent, as long as the regularization maintains gauge-invariance.

Unfortunately, life is not so simple. In fact, in an influential paper [5], Hoodbhoy, Ji, and Lu claimed that the gluon spin evolves differently in two gauges, i.e. the LC gauge and the Feynman gauge (a typical covariant gauge). According to them, the calculation in the Feynman gauge does not reproduce the well-known Altarelli-Parisi evolution equation for $\Delta\Sigma$ and Δg [18]. Undoubt-

edly, the difficulty is connected with the widely-known observation that there is no gauge-invariant twist-2 local operator corresponding to the gluon spin. A state of disorder has been strengthened further by the recent claim by Wong et al. [14]. (See also a similar assertion by Cho et al. for the evolution of quark and gluon momentum fractions [19].) They claim that the gluon spin in their decomposition evolves differently from the standard Altarelli-Parisi equation. This sounds strange to us, because their definition of gluon spin can be thought of as a gauge-fixed form of our more general (or formal) gauge-invariant expression given in the paper [12]. If gauge-invariance is maintained at every stage of manipulation, the answer should be the same as that obtained in the LC gauge.

Now, the purpose of the present paper is to resolve the above-explained puzzle. To this end, we shall explicitly calculate the 1-loop anomalous dimension of the quark and gluon spin operators appearing in our decomposition within the Feynman gauge and show that the answer precisely coincides with that obtained in the LC gauge.

Among the four terms in the decomposition [12], we concentrate here on the quark and gluon spin operators given by

$$M^{+12}_{q-spin} = \bar{\psi}\,\gamma^+\,\gamma_5\,\psi, \tag{1}$$

$$M^{+12}_{g-spin} = 2\,\mathrm{Tr}\left[\,F^{+1}\,A^2_{phys} \;-\; F^{+2}\,A^1_{phys}\,\right]. \tag{2}$$

The gauge-invariance of M^{+12}_{q-spin} is self-evident, while that of M^{+12}_{g-spin} is ensured by the covariant transformation property of the *physical* part of the gluon field under a gauge transformation, i.e. $A^\mu_{phys}(x) \rightarrow U(x)\,A^\mu_{phys}(x)\,U^{-1}(x)$.

We start with writing down a little more explicit form of our gluon spin operator :

$$M^{+12}_{g-spin} \;=\; V_A \;+\; V_B \;+\; V_C, \tag{3}$$

with

$$V_A = (\partial^+ A^1_a)\,A^2_{a,phys} \;-\; (\partial^+ A^2_a)\,A^1_{a,phys}, \tag{4}$$

$$V_B = -\,\left[\,(\partial^1 A^+_a)\,A^2_{a,phys} \;-\; (\partial^2 A^+_a)\,A^1_{a,phys}\,\right], \tag{5}$$

$$V_C = g\,f_{abc}\,A^+_b\,(\,A^1_c\,A^2_{a,phys} \;-\; A^2_c\,A^1_{a,phys}\,). \tag{6}$$

In the LC gauge ($A^+_a = 0$), which falls into the category called the physical gauge, only the vertex V_A survives that simplifies the work process significantly [20]. Note however that, in general covariant gauge including the Feynman gauge, the gluon spin operator is not given by V_A alone, but it is a sum of the three pieces V_A, V_B and V_C. We point out that V_B and V_C have basically the same form as the operators $O_1 = -\int d^3x\,\nabla A^+_a \times \vec{A}_a$ and $O_2 = -\int d^3x\,f_{abc}\,A^+_c \vec{A}_b \times \vec{A}_a$, which, together with the naive gluon spin operator S^+_g in the LC gauge, were considered in the Feynman gauge calculation of the gluon spin evolution equation [5]. However, there is a very delicate difference between V_A, V_B, V_C in Eqs.(4)-(6) and S^+_g, O_1, O_2 considered in [5]. One should notice that one of the gluon fields in our operators V_A, V_B, V_C is its physical component. (Note that this is an important factor, which ensures the gauge-invariance of our gluon spin operator.)

The question is then how to introduce this unique feature of our gluon spin operator into the Feynman rules for evaluating corresponding anomalous dimensions. Remember first that the gluon propagator in general covariant gauge is given by

$$\begin{aligned} D^{\mu\nu}_{ab}(k) &= \frac{i\,\delta_{ab}}{k^2 + i\epsilon}\,\sum_{\lambda=1}^{4}\,\varepsilon^\mu(k,\lambda)\,\varepsilon^\nu(k,\lambda) \\ &= \frac{i\,\delta_{ab}}{k^2 + i\varepsilon}\,\left(-\,g^{\mu\nu} \;+\; (1-\xi)\,\frac{k^\mu k^\nu}{k^2 + i\varepsilon}\right), \end{aligned} \tag{7}$$

with ξ being an arbitrary gauge parameter. The Feynman gauge corresponds to taking $\xi = 1$, while the Landau gauge to $\xi = 0$. Since one of the gluon field appearing in our gluon spin operator is its physical part, we must replace the gluon propagator by

$$\frac{i\,\delta_{ab}}{k^2 + i\epsilon}\,\sum_{\lambda=1}^{2}\,\varepsilon^\mu(k,\lambda)\,\varepsilon^\nu(k,\lambda), \tag{8}$$

when one of the endpoint of gluon propagator is obtained by the contraction with the physical part of A_μ in our gluon spin operator. Here, we need a sum of the product of gluon polarization vector over *two physical polarization states* (not including the scalar and longitudinal polarization). The answer is well-known [21],[22]. Given below is its covariant form, which holds in an arbitrary Lorentz frame :

$$\begin{aligned} T^{\mu\nu} &\equiv \sum_{\lambda=1}^{2}\,\varepsilon^\mu(k,\lambda)\,\varepsilon^\nu(k,\lambda) \\ &= -\,g^{\mu\nu} \;+\; \frac{k^\mu\,n^\nu + n^\mu\,k^\nu}{n\cdot k} \;-\; n^2\,\frac{k^\mu\,k^\nu}{(n\cdot k)^2}, \end{aligned} \tag{9}$$

where n being an arbitrary four-vector subject to the condition $n\cdot\varepsilon = 0$ and $n\cdot k \neq 0$. For practical calculation, it is convenient to take n to be a lightlike four-vector with $n^2 = 0$. In this case, the modified gluon propagator reduces to

$$\begin{aligned} \tilde{D}^{\mu\nu}_{ab}(k) &\equiv \frac{i\,\delta_{ab}}{k^2 + i\epsilon}\,\sum_{\lambda=1}^{2}\,\varepsilon^\mu(k,\lambda)\,\varepsilon^\nu(k,\lambda) \\ &= \frac{i\,\delta_{ab}}{k^2 + i\epsilon}\,\left(-\,g^{\mu\nu} \;+\; \frac{k^\mu\,n^\nu + n^\mu\,k^\nu}{n\cdot k}\right), \end{aligned} \tag{10}$$

which precisely coincides with the gluon propagator in the LC gauge. This does not mean that we are working in the LC gauge from the very beginning. In fact, if we did so, there would be no contributions to the anomalous dimensions from the operators V_B and V_C. As pointed

out before, it is crucial to use the above propagator only when one of the endpoint of the gluon propagator is obtained by the contraction with the physical part of A_μ in our gluon spin operator. In other places, we should use the standard gluon propagator, which, for instance in the Feynman gauge, is given by

$$D_{ab}^{\mu\nu}(k) \;=\; \frac{i\,\delta_{ab}}{k^2 + i\epsilon}\,(\,-\,g^{\mu\nu}\,)\,. \tag{11}$$

The momentum space vertex operators for the gluon spin, which takes account of the above subtlety, can be expressed by the following formulas supplemented with the diagrams shown in Fig.1 :

$$\begin{aligned} V_A &= i\,k^+\,(\,g^{\mu 1}\,g^{\nu 2} \,-\, g^{\mu 2}\,g^{\nu 1}\,)\,P_T^\nu\,\delta_{ab} \\ &\quad -\;(\mu \leftrightarrow \nu), \end{aligned} \tag{12}$$

$$\begin{aligned} V_B &= -\,i\,g^{\mu +}\,(\,k^1\,g^{\nu 2} \,-\, k^2\,g^{\nu 1}\,)\,P_T^\nu\,\delta_{ab} \\ &\quad -\;(\mu \leftrightarrow \nu), \end{aligned} \tag{13}$$

$$\begin{aligned} V_C &= g\,f_{abc}\,g^{\lambda +}\,(\,g^{\mu 1}\,g^{\nu 2} \,-\, g^{\mu 2}\,g^{\nu 1}\,)\,(\,P_T^\mu \,+\, P_T^\nu\,) \\ &\quad +\, g\,f_{abc}\,g^{\mu +}\,(\,g^{\nu 1}\,g^{\lambda 2} \,-\, g^{\nu 2}\,g^{\lambda 1}\,)\,(\,P_T^\nu \,+\, P_T^\lambda\,) \\ &\quad +\, g\,f_{abc}\,g^{\nu +}\,(\,g^{\lambda 1}\,g^{\mu 2} \,-\, g^{\lambda 2}\,g^{\mu 1}\,)\,(\,P_T^\lambda \,+\, P_T^\mu\,). \end{aligned} \tag{14}$$

Here, P_T^ν is a kind of projection operator, which reminds us of the fact that we must use the gluon propagator $\tilde{D}_{ab}^{\mu\nu}(k)$ given by (10), whenever it contains the Lorentz index ν.

Keeping in mind somewhat nonstandard Feynman rule explained above, we are now ready to calculate the 1-loop anomalous dimension of the quark and gluon spin operators. The graphs $(a), (b), (c)$ and (d), shown in Fig.2, respectively contribute to the 1-loop anomalous dimensions $\Delta\gamma_{qq}^{(0)}, \Delta\gamma_{qG}^{(0)}, \Delta\gamma_{Gq}^{(0)}$, and $\Delta\gamma_{GG}^{(0)}$. Graphs with external self-energies are not shown in the figure. Graphs which are not symmetric with respect to the vertical lines through the operator vertex have to be counted twice.

For the quark spin operator, we obtain just the expected answer :

$$\Delta\gamma_{qq}^{(0)} = \frac{\alpha_S}{2\,\pi}\cdot\frac{1}{2}\,C_F \;+\; \frac{\alpha_S}{2\,\pi}\cdot\left(-\,\frac{1}{2}\,C_F\right) \;=\; 0, \tag{15}$$

$$\Delta\gamma_{qG}^{(0)} = \quad 0, \tag{16}$$

with $\alpha_S = g^2/(4\,\pi)$ and $C_F = 4/3$. Here, the 2nd term of $\Delta\gamma_{qq}^{(0)}$ comes from the quark field-strength renormalization in the Feynman gauge. For the gluon spin part, we have

$$\Delta\gamma_{Gq}^{(0)} = \frac{\alpha_S}{2\,\pi}\cdot C_F \;+\; \frac{\alpha_S}{2\,\pi}\cdot\frac{1}{2}\,C_F \;=\; \frac{\alpha_S}{2\,\pi}\cdot\frac{3}{2}\,C_F, \tag{17}$$

where the 1st term is the contribution of the vertex V_A, while the 2nd is that of V_B. Finally, we find that

$$\begin{aligned} \Delta\gamma_{GG}^{(0)} &= \frac{\alpha_S}{2\,\pi}\cdot\frac{11}{24}\,C_A \;+\; \frac{\alpha_S}{2\,\pi}\cdot\left(-\,\frac{23}{24}\,C_A\right) \\ &\quad +\, \frac{\alpha_S}{2\,\pi}\cdot\frac{3}{2}\,C_A \;+\; \frac{\alpha_S}{2\,\pi}\cdot\left(\frac{5}{6}\,C_A - \frac{1}{3}\,n_f\right) \\ &= \frac{\alpha_S}{2\,\pi}\cdot\left(\frac{11}{6}\,C_A - \frac{1}{3}\,n_f\right), \end{aligned} \tag{18}$$

with $C_A = 3$ and n_f being the number of quark flavors. Here, the 1st, 2nd, and the 3rd terms are respectively the contributions from the operators V_A, V_B and V_C, whereas the 4th term comes from the gluon field-strength renormalization in the Feynman gauge.

One confirms that the final answer just coincides with the lowest-order anomalous dimensions corresponding to the longitudinally polarized quark and gluon distributions obtained with the Altarelli-Parisi method. We emphasize that, in the standard operator expansion framework, a direct confirmation of the above answer was done only in the LC gauge, because there is no gauge-invariant local operator corresponding to the gluon spin. As a consequence, the Altarelli-Parisi evolution equation for the 1st moment of $\Delta q(x)$ and $\Delta g(x)$ has been justified only on the basis of extrapolation or analytic continuation of higher anomalous dimensions. Our calculation shows that, although nonlocal, Eq.(3)-(6) in fact gives the gauge-invariant operator definition of the gluon spin, which reproduces the Altarelli-Parisi evolution equation even in the Feynman gauge. We can further show that the final answer for the 1-loop anomalous dimensions is exactly the same also in other covariant gauges containing an arbitrary gauge parameter ξ. Here, the dependence on the gauge parameter appears in the graph (a) and the left graph of (d), but it is precisely cancelled by the ξ-dependent terms arising from the quark and gluon field-strength renormalization. In this way, the gauge-invariance of our gluon spin operator with quantum-loop corrections is established.

In conclusion, we have calculated the 1-loop anomalous dimensions of the gluon spin operator appearing in our gauge-invariant decomposition of the nucleon spin [12]. We find that the answer is in fact independent of the choice of gauge. The answer obtained in the Feynman gauge (or any covariant gauges) just reproduces the answer obtained in the LC gauge, which is also the answer of the famous Altarelli-Parisi method. The key factor leading to the correct answer is the proper sum over the gluon polarization states appearing in the Feynman rule. Note that such delicacy does not arise in the Altarelli-Parisi method, in which only real splitting processes containing physical gluons alone (not virtual gluons) come into the calculation. At any rate, we have now confirmed that the gluon spin operator appearing in our nucleon spin decomposition provides us with the long-coveted operator definition of gluon spin, which precisely reproduces the same evolution equation as obtained by the Altarelli-Parisi method in an arbitrary gauge. This means that, in combination with the previous two papers, we have solved the long-standing puzzle concerning the gauge-invariant decomposition of the nucleon spin.

This work is supported in part by a Grant-in-Aid for

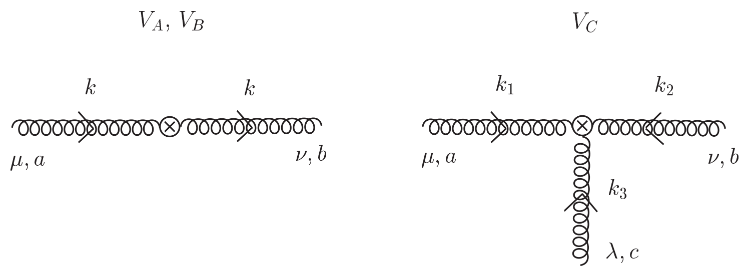

FIG. 1. Momentum space vertices for the gluon spin operator.

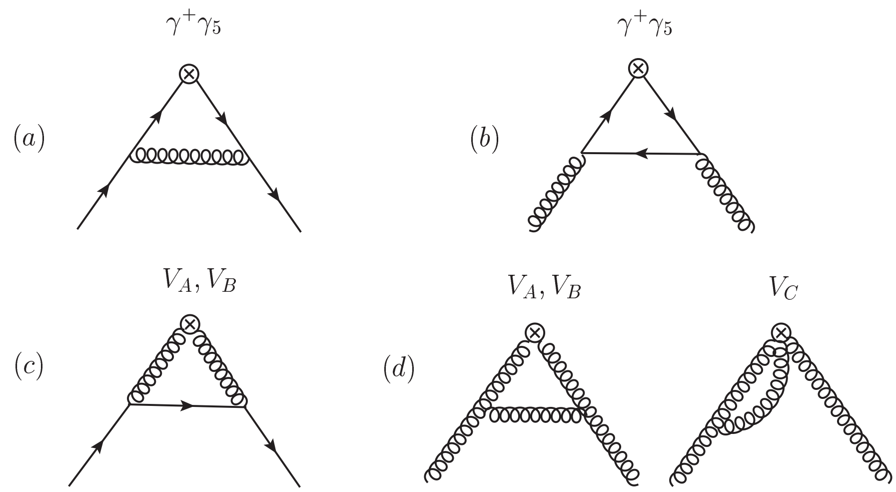

FIG. 2. One-loop graphs contributing to the anomalous dimensions of quark and gluon spin operators : (a) $\Delta\gamma^{(0)}_{qq}$, (b) $\Delta\gamma^{(0)}_{qG}$, (c) $\Delta\gamma^{(0)}_{Gq}$, and (d) $\Delta\gamma^{(0)}_{GG}$. Graphs with external self-energies are not shown in the figure. Graphs that are not symmeric with respect to the vertical lines through the operator vertex have to be counted twice.

Scientific Research for Ministry of Education, Culture, Sports, Science and Technology, Japan, Grant No. C-40135653.

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